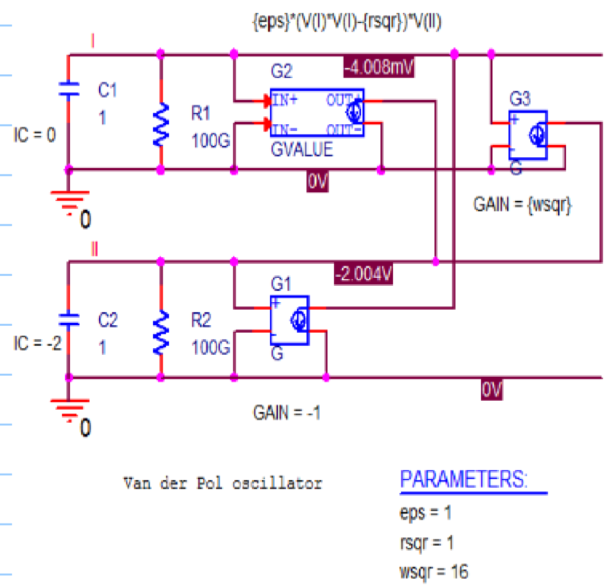
ENEE 610
09/22/20

Note Title 8/31/2020

Homework 3. Also a good base paper topic may be Adiabatic Logic Circuits; a review paper is by V. I. Starosel'skii on the web.

Van der Pol Oscillator circuit. Passivity S matrix properties, all pass, circulator, loaded circulator

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netlist:
* source VANDERPOL
C_C1        I 0  1 IC=0 TC=0,0
C_C2        II 0  1 IC=-2 TC=0,0
G_G1        I 0 II 0 -1
R_R1        I 0  100G TC=0,0
R_R2        II 0  100G TC=0,0
G_G2        II 0 VALUE {
{eps}*(V(I)*V(I)-{rsqr})*V(II) }
G_G3        II 0 I 0 {wsqr}
.PARAM  eps=1 rsqr=1 wsqr=16
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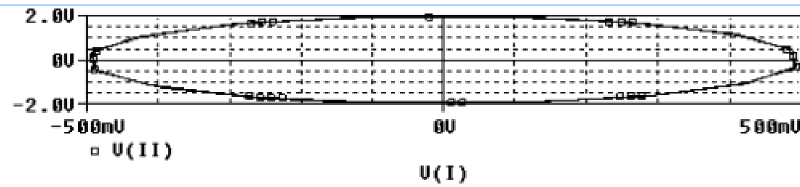
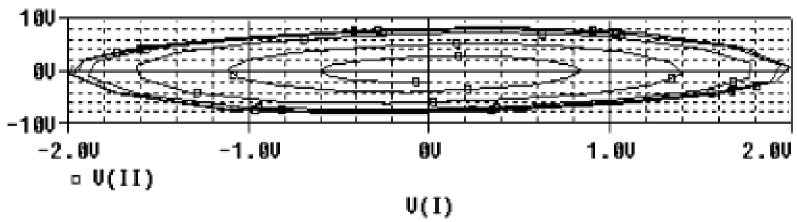
ODE in circuits

$$\ddot{x} + \epsilon(x^2 - r^2)\dot{x} + \omega_0^2 x = 0 \quad , \quad x(0),\ \dot{x}(0) \quad \text{Van der Pol oscillator}$$

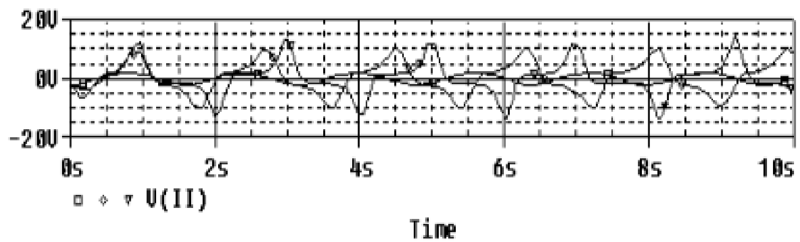
$$\Rightarrow \begin{cases} \dot{x} = y \\ \dot{y} = \ddot{x} = -\epsilon(x^2 - r^2)y - \omega_0^2 x \end{cases} \Big\} \text{ state variable equations}$$



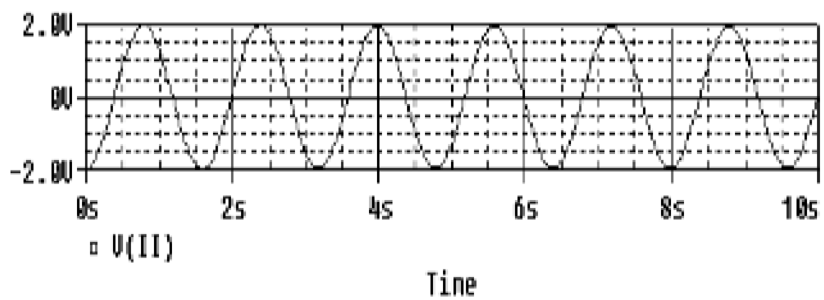
ICs: C1=>0,C2=>-2
eps=0



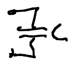
ICs: C1=>0,C2=>-2
eps=1



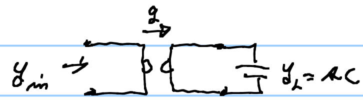
eps = 0, 5 10



eps=0

Ex:  $y_c = sC$, $S_c(s) = (1+sC)^{-1}(1-sC) = \frac{1-sC}{1+sC}$ $\quad s=\sigma+j\omega$

$S_c(j\omega) = \frac{1-j\omega C}{1+j\omega C}$; $|S_c(j\omega)| = \frac{\sqrt{1+(\omega C)^2}}{\sqrt{1+(\omega C)^2}} = 1$

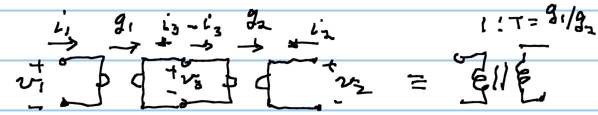


$L = \frac{C}{g^2}$

$Y = \begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix}$

$y_{in}(s) = \frac{y_{11}y_L + \Delta Y}{y_{22}+y_L} = \frac{0+g^2}{sC} = \frac{1}{s(C/g^2)} = \frac{1}{sL}$

$\Rightarrow \equiv$  $L = g^2/C$



$1 : T = g_1/g_2$

$i_1 = -g_1 v_3 \qquad -i_3 = -g_2 v_2$

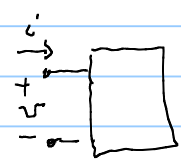
$g_2 v_2 = i_3 = g_1 v_1 \qquad i_2 = g_2 v_3 = g_2\left(\frac{-i_1}{g_1}\right)$

$v_2 = T v_1 \qquad v_2 = \left(\frac{g_1}{g_2}\right) v_1 \qquad i_1 = \left(-g_1/g_2\right) i_2$

$i_1 = -T i_2$

$p(t) = (v_1 i_1 + v_2 i_2) = v^T(t)\, i(t) = (v_1[-T] i_2 + (T v_1) i_2) = 0$ lossless in time

Passivity $\quad \mathcal{E}(t) = \int_{-\infty}^{t} p(\tau)\, d\tau \quad$ If $\mathcal{E}(t) \geq 0$ for every t, including $t=\infty$



$p(t) = v^T i = \frac{v^T i + i^T v}{2}$

$j = \sqrt{-1} \Rightarrow -j$ complex conjugate

$\mathcal{E}(\infty) = \int_{-\infty}^{\infty} v^T i \, d\tau \Rightarrow$ Parseval's theorem $\quad \int_{-\infty}^{\infty} v_i^*(t)\, i_i(t)\, dt = \int_{-\infty}^{\infty} V_i^*(j\omega)\, I_i(j\omega)\, df$

power equivalents

$\int_{-\infty}^{\infty} \left(\frac{v^T(\tau) i(\tau) + i^T(\tau) v(\tau)}{2}\right) d\tau = \int_{-\infty}^{\infty} \frac{V^{T*}(j\omega) I(j\omega) + I^{T*}(j\omega) V(j\omega)}{2}\, d\frac{2\pi f}{2\pi} = \omega$

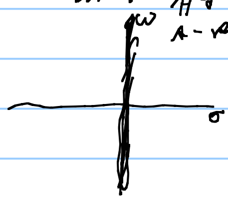
$$= \int_{-\infty}^{\infty} \left(\frac{V^{T*}(j\omega) Y(j\omega) V(j\omega) + \left(V^{T*}(j\omega) Y^{T*}(j\omega) V(j\omega)\right)}{2} \right) \frac{d\omega}{2\pi}$$

$$= \int_{-\infty}^{\infty} V^{T*}(j\omega) \left[\frac{Y(j\omega) + Y^{T*}(j\omega)}{2} \right] V(j\omega) \frac{d\omega}{2\pi} = \int_{-\infty}^{\infty} V^{T*}(j\omega) \underbrace{Y}_{\text{Hermitian}}(j\omega) V(j\omega) \frac{d\omega}{2\pi}$$

for all $V(j\omega)$ gives a power constraint $Y_H(j\omega) = \dfrac{Y(j\omega) + Y^{T*}(j\omega)}{2}$

If the circuit is passive with a $Y(s)$

then $Y_H(j\omega) \Rightarrow \mathcal{E}(\infty) \geq 0 \Rightarrow Y_H(j\omega)$ is positive semi-definite



s-plane, $s = \sigma + j\omega$

for $\sigma > 0$ expect no singularity by stability

If $Y_{ij}(s) = \dfrac{n(s)_{ij}}{d(s)_{ij}} = \dfrac{a_m s^m + a_{m-1} s^{m-1} + \dots + a_1 s + a_0}{d_m s^m + \dots + d_1 s + d_0}$

$Y_{ij}(s) = Y_{ij}(j\omega)\Big|_{j\omega = s}$ by analytic continuation we know for rational functions by $\omega = s/j$

for a passive circuit

$$\left.\begin{array}{l} Y(s) \text{ has no singularities in } \sigma > 0 \\ \left.\dfrac{Y(s) + Y^T(-s)}{2}\right|_{s=j\omega} \text{ is positive semidefinite} \\ Y(s) \text{ is real for real } s,\ s = \sigma > 0 \end{array}\right\} \text{positive real matrix}$$

If $Y(s)$ doesn't exist & the circuit is passive then $S(s)$ exists

Power for scattering $\quad \mathcal{E}(t) = \displaystyle\int_{-\infty}^{t} \frac{v^T i + i^T v}{2} d\tau \; ; \quad v = v^i + v^r, \quad i = v^i - v^r$

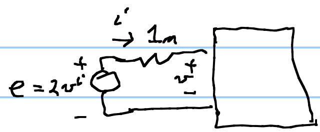
$v^T i = (v^i + v^r)^T (v^i - v^r) = v^{iT} v^i - v^{iT} v^r + v^{rT} v^i - v^{rT} v^r$

$i^T v = (v^i - v^r)^T (v^i + v^r) = \quad '' \quad + v^{iT} v^r - v^{rT} v^i - \quad ''$

$$\mathcal{E}(\infty) = \int_{-\infty}^{t=\infty} \left[v^{iT} v^{i} - v^{rT} v^{r} \right] d\tau = \int_{-\infty}^{\infty} \left(V^{iT*} V^{i} - V^{rT*} \cdot V^{r} \right) \frac{d\omega}{2\pi}$$

$$V^{r}(s) = S(s)\, V^{i}(s) \qquad = \int_{-\infty}^{\infty} V^{iT*} \left[1_m - S^{T*}(j\omega) S(j\omega) \right] V^{i} \frac{d\omega}{2\pi}$$

$$\left.\begin{array}{l} \text{for passive } 1_m - S^{T*}(j\omega) S(j\omega) \text{ is positive semidefinite} \\ \text{now still no poles but in } \sigma \geq 0 \\ \& \ S(\sigma) \text{ is real in } \sigma > 0 \end{array}\right\} \text{Bounded Real } S$$



$$\underbrace{\int_{-\infty}^{\infty} v^{iT*}(j\omega)\, v^{i}(j\omega) \frac{d\omega}{2\pi}}_{\geq 0} = \int_{-\infty}^{\infty} \Big(V^{T*}(j\omega) V(j\omega) + I^{T*}(j\omega) I(j\omega) + \underbrace{2 V^{T*}(j\omega) I(j\omega)}_{\geq 0} \Big) \frac{d\omega}{2\pi}$$

if passive

$$\Rightarrow \int_{-\infty}^{\infty} (V^{T*} V + I^{T*} I)\, d\omega \geq 0 \ \ \& \ S \text{ will exist}$$